Employee Stock Option Exercise and Firm Cost

The Journal of Finance
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00 Introduction

- The valuation of long-maturity American options such as ESOs is sensitive to assumptions about how they will be exercised.
- Commonly used modifications to Black-Scholes (1973) are inadequate because they ignore key empirical features of employee exercise patterns.

In this paper, we

1. estimate the rate of voluntary exercise as a function of the stock price path and of firm, contract, and option holder characteristics.
2. develop an analytic valuation approximation, compare option values calculated using our model and Black-Scholes-based methods commonly used in practice.

01 Previous Literature

The employee chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions such as portfolio, consumption, and effort choice, and this typically leads to some early exercise for the purpose of diversification.

A number of analytic methods for approximating employee option value have also been proposed.

- FAS 123R permits using the BS formula with the expiration date replaced by the option's expected life, and SAB 110 permits using BS with expiration replaced by the average of the contractual vesting date and expiration date.
- Black-Scholes-based approximation:
  Jennergren and Näslund (1993)、Carr and Linetsky (2000)、
  Cvitanić,Wiener, and Zapatero (2008)

02 Valuation and Estimation Methodology

1. The fraction of remaining vested in-the-money options held by individual i from grant j that is exercised on day t :

$$y_{ijt} = G(X_{ijt}\beta) + \epsilon_{ijt}$$

where $G(X_{ijt}\beta) = \frac{\exp(X_{ijt}\beta)}{1+\exp(X_{ijt}\beta)}$, I_t is the information set at date t,

X_{ijt} is a set of covariates in I_t ,

$E(\epsilon_{ijt}|I_t) = 0$ and $E(\epsilon_{ijt}, \epsilon_{i'j't'}) = 0$ for $i \neq i'$ or $t \neq t'$

The exercise rate function $G(X_{ijt}\beta)$ is the probability that an individual option in the grant will be exercised at date t, conditional on the time, stock price, and other state variables described by X_{ijt}, and conditional on having survived to time t.

For unvested options, and for vested options that are out of the money, we define $G \equiv 0$.

02 Valuation and Estimation Methodology

2. The average option value (the cost of the option to the firm) is

$$O_t = \sum_{k=1}^{n} \alpha_k \mathrm{E}_t^* \left\{ \int_{t \vee t_k}^{T} e^{-r(\tau - t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s + \lambda)\, ds}\, d\tau \right.$$

$$\left. + e^{-r(T-t)} e^{-\int_t^T (G_s + \lambda)\, ds} (S_T - K)^+ \right\}.$$

where λ describe the probability that the option will be involuntarily stopped through termination, forcing exercise of vested in-the-money options, cancellation of vested out-of-the-money options, or forfeiture of unvested options.

03 Data and Variable Construction

Data

- Time : 1981~2009
- 88 publicly traded corporations (27 of them are S&P500 firms)
- over 290,000 employees
- over 810,000 employee option grants
- more than 560,000 different exercise events
- complete histories of ESO grants, vesting structures, and option exercise, cancellation, and termination events for all employees

03 Data and Variable Construction

Table II

Summary Statistics for Employee Option Grants, Vesting Structures, and Exercises

This table provides summary statistics within industrial groupings for employee option grants, vesting structures, and exercise patterns.

SIC 3 : technology firms

	Construction and Manufacturing		Transportation, Communication, and Utilities	Retail	Finance, Insurance, and Real Estate	Services		All
One-Digit SIC	1 and 2	3	4	5	6	7	8 and 9	
Panel A: Aggregate Number of Employees, Grants, and Exercise Events								
Number of employees	5,443	75,102	11,955	34,220	110,680	48,385	6,267	292,052
Number of grants	20,584	249,190	18,736	116,955	254,269	137,086	13,576	810,348
Number of exercises	16,947	146,183	5,366	99,142	179,615	106,255	7,595	561,073
Panel B: Number of Grants per Employee								
Mean	3.78	3.33	1.56	3.42	2.97	2.83	2.16	2.17
Median	3.00	2.00	1.00	2.00	1.00	2.00	2.00	1.00
SD	3.68	1.41	1.41	3.19	2.36	2.98	2.00	2.01
Panel C: Dollar Value of Underlying Shares per Grant ($ Thousands)								
Mean	90.27	46.15	106.32	52.64	106.41	59.34	40.05	65.51
Median	25.56	13.65	52.32	11.51	25.85	26.89	11.60	16.65
SD	379.99	263.48	321.81	299.67	715.94	243.27	158.38	425.66
Panel D: Number of Vesting Dates per Grant								
Mean	9.25	6.90	1.30	2.70	2.95	4.36	3.14	4.38
Median	4.00	4.00	1.00	2.00	3.00	4.00	2.00	3.00
SD	14.55	9.30	0.86	1.84	1.36	3.42	1.25	6.52

03 Data and Variable Construction

Table II—*Continued*

SIC 3 : technology firms

	Construction and Manufacturing		Transportation, Communication, and Utilities	Retail	Finance, Insurance, and Real Estate	Services		All
One-Digit SIC	1 and 2	3	4	5	6	7	8 and 9	
Panel E: Percentage of Options That Vest on the First Vesting Date (per Grant)								
Mean	42.77	37.16	90.38	61.69	48.32	38.54	37.63	51.28
Median	25.00	25.00	25.00	33.33	33.33	25.00	50.00	33.33
SD	31.82	30.48	20.67	38.64	29.54	29.75	14.75	35.63
Panel F: Number of Months from Grant to Full Vesting (per Grant)								
Mean	36.39	44.76	44.44	28.13	33.80	40.49	37.74	56.93
Median	48.00	48.00	48.00	36.00	36.00	48.00	24.00	38.00
SD	15.78	20.34	5.48	21.31	19.75	20.87	15.72	25.56
Panel G: Number of Months from Grant to Expiration (per Grant)								
Mean	113.06	72.00	120.00	122.40	120.18	120.00	120.00	120.29
Median	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
SD	19.81	5.48	0.00	12.63	0.38	0.00	0.00	6.42
Panel H: Fraction of Grant's Vested Options That Are Exercised (per Exercise Event)								
Mean	0.72	0.84	0.58	0.68	0.58	0.57	0.93	0.67
Median	1.00	1.00	0.67	0.81	0.62	0.51	1.00	0.95
SD	0.35	0.29	0.22	0.35	0.40	0.39	0.18	0.38

03 Data and Variable Construction

ESO Exercise Factors

1. Standard Option Exercise Factors

- Price-to-strike ratio : $\frac{adjusted\ stock\ price}{strike\ price} - 1$
- Volatility : The annualized volatility of the daily stock return, estimated over the prior three months.
- Dividend in next two weeks : $1_{\{dividend\ will\ be\ paid\ in\ 2\ weeeks\}} \times \frac{dividend\ payment}{current\ stock\ price}$
- Years to expiration : The number of years remaining until the expiration date of the grant.

03 Data and Variable Construction

ESO Exercise Factors

2. Portfolio Factors

- Correlation : The correlation between the stock return and the return on the S&P 500 Composite Index.
- Black-Scholes employee option risk : Approximated as the total Black-Scholes delta of the employee's option portfolio times the dollar volatility of the stock price.
- Black-Scholes employee option wealth : The log of the total Black-Scholes value of the employee's option portfolio.

03 Data and Variable Construction

ESO Exercise Factors

2. Portfolio Factors
Based on a theoretical analysis of the optimal exercise boundary, they show that, as with a standard American call, exercise is more likely the greater is the dividend rate. However, unlike a standard American call:

(1) exercise is more likely if the employee has greater absolute risk aversion;
(2) exercise is less likely with greater employee wealth, provided the employee has decreasing absolute risk aversion;
(3) exercise is decreasing in the absolute magnitude of the correlation between the stock return and the market return;
(4) the effect of greater volatility or longer time to expiration is ambiguous. (due to the conflicting effects of employee risk aversion and the convexity of the option payoff.)

03 Data and Variable Construction

ESO Exercise Factors

2. Portfolio Factors
In standard option theory, variables such as volatility and time to expiration only affect exercise behavior when the underlying stock pays dividends. However, in the utility maximization framework for nontransferable options, these variables can affect exercise behavior even in the absence of dividends, though the sign of the effect is theoretically ambiguous.
在一般的選擇權理論中，波動度及到期日時間長僅在有發股利時才會影響履約行為；但在不可轉讓選擇權之效用最大化的框架中，即便不發股利，此兩變因皆會影響履約行為

03 Data and Variable Construction

ESO Exercise Factors

3. Additional Factors

- Vesting date in past two weeks : A dummy indicating whether a vesting date has occurred in the previous two weeks for the given grant.
- Vesting date in past two weeks × years between prior two vesting dates
- Price ≥ 90th percentile of prior-year distribution
- Top-10 option holders
- Executive : An indicator of whether the employee is a senior executive.
- Male
- Age

04 Estimation Results

The dependent variable is the fraction of remaining vested options exercised by an employee from a given grant on a given trading day.

$$y_{ijt} = G\left(X_{ijt}\beta\right) + \epsilon_{ijt}$$

	Standard Factors	Standard and Portfolio Factors	Standard, Portfolio, and Additional Factors		
			Top-10 Holders		Executive
	(1)	(2)	(3)	(4)	(5)
Standard factors					
Constant	−7.5240	−7.7470	−8.8192	−7.9787	−9.9165
	(0.0102)	(0.0196)	(0.0198)	(0.0638)	(0.1262)
Price-to-strike ratio	0.0080	0.0080	0.0070	0.0080	0.0217
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0021)
Volatility	−0.3217	−0.5492	−0.2460	−0.6664	0.2005
	(0.0138)	(0.0205)	(0.0206)	(0.0304)	(0.0996)
Div next two weeks	−4.6165	−4.4559	1.2100	9.1965	7.3247
	(0.5038)	(0.4973)	(0.1668)	(0.2599)	(1.5831)
Years to expiration	0.0840	0.0855	0.0211	−0.0478	−0.3237
	(0.0011)	(0.0011)	(0.0011)	(0.0017)	(0.0047)
Portfolio factors					
Correlation		0.1855	0.4564	1.1923	1.0295
		(0.0136)	(0.0139)	(0.0194)	(0.0406)
BS Empl. option risk		0.0980	0.2727	0.2701	0.2366
		(0.0079)	(0.0071)	(0.0086)	(0.0312)
BS Empl. option wealth		−0.0936	−0.2605	−0.2536	−0.1890
		(0.0080)	(0.0072)	(0.0087)	(0.0310)
Additional factors					
Vest past two weeks			1.9964	2.5820	3.9163
			(0.0089)	(0.0091)	(0.0316)
Vest past two weeks × years bet. prior two vest dates			0.9280	0.4736	0.9577
			(0.0041)	(0.0042)	(0.0132)
Price ≥ 90th percentile of prior-year distribution			1.1691	1.0424	1.1387
			(0.0056)	(0.0069)	(0.0136)
Top-10 option holders/executive			−0.2455	−0.2325	−0.0898
			(0.0165)	(0.0229)	(0.0182)
Male				0.2089	0.0676
				(0.0072)	(0.0144)
Age				−0.0053	0.0718
				(0.0025)	(0.0052)
Age^2				−0.0000	−0.0008
				(0.0000)	(0.0001)

04 Estimation Results

The dependent variable is the fraction of remaining vested options exercised by an employee from a given grant on a given trading day.

$$y_{ijt} = G\left(X_{ijt}\beta\right) + \epsilon_{ijt}$$

- Vesting past two weeks: the notification effect is larger for lower ranked employees, while the pent-up demand effect is larger for top employees.

	Top-10, 59-Firm Sample 239M Observations Specification (6)			Executive, 22-Firm Sample 59M Observations Specification (7)		
	Lower	Top	Diff	Lower	Top	Diff
Standard Factors						
Constant	−8.0505	−8.3923	−0.3418	−9.6167	−9.9842	−0.3676
	(0.0650)	(0.5899)	(0.5930)	(0.1322)	(0.4731)	(0.4906)
Price-to-strike ratio	0.0082	0.0054	−0.0027	0.0169	0.0272	0.0103
	(0.0002)	(0.0005)	(0.0005)	(0.0024)	(0.0043)	(0.0049)
Volatility	−0.8511	0.3595	1.2106	1.0220	−3.2749	−4.2969
	(0.0392)	(0.1133)	(0.1198)	(0.1039)	(0.2682)	(0.2845)
Div next two weeks	10.3836	−9.5760	−19.9595	6.3412	11.2023	4.8610
	(0.2829)	(11.2650)	(11.2684)	(1.7331)	(3.8949)	(4.2387)
Years to expiration	−0.0473	−0.1130	−0.0657	−0.3645	−0.1290	0.2355
	(0.0018)	(0.0123)	(0.0128)	(0.0053)	(0.0097)	(0.0110)
Portfolio factors						
Correlation	1.1774	1.1468	−0.0306	1.1425	0.9411	−0.2014
	(0.0196)	(0.1242)	(0.1255)	(0.0442)	(0.1004)	(0.1071)
BS Empl. option risk	0.3394	−0.0697	−0.4091	−0.0698	1.5971	1.6669
	(0.0135)	(0.0321)	(0.0348)	(0.0351)	(0.0721)	(0.0802)
BS Empl. option wealth	−0.3256	0.0997	0.4253	0.1230	−1.6129	−1.7360
	(0.0139)	(0.0313)	(0.0342)	(0.0349)	(0.0732)	(0.0810)
Additional factors						
Vest past two weeks	2.5885	2.3019	−0.2867	4.1218	3.0223	−1.0995
	(0.0091)	(0.0821)	(0.0826)	(0.0359)	(0.0598)	(0.0696)
Vest past two weeks × years bet. prior two vest dates	0.4676	0.7325	0.2649	0.9018	1.2146	0.3128
	(0.0042)	(0.0394)	(0.0396)	(0.0144)	(0.0270)	(0.0306)
Price ≥ 90th percentile of prior-year dist'n	1.0493	0.8312	−0.2181	1.1653	1.0243	−0.1410
	(0.0070)	(0.0457)	(0.0463)	(0.0151)	(0.0308)	(0.0342)
Male	0.2108	0.1228	−0.0880	0.0213	0.2485	0.2271
	(0.0072)	(0.0889)	(0.0892)	(0.0158)	(0.0375)	(0.0407)
Age	−0.0029	0.0149	0.0178	0.0711	0.0329	−0.0382
	(0.0026)	(0.0214)	(0.0216)	(0.0055)	(0.0192)	(0.0199)
Age^2	−0.0001	−0.0001	−0.0001	−0.0008	−0.0004	0.0003
	(0.0000)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)

04 Estimation Results

Table V

Change in Exercise Rate for One-Standard-Deviation Increase in Exercise Factors

This table presents the percentage change in the estimated exercise rate G for one-standard-deviation increase in the exercise factors, relative to the exercise rate evaluated at the sample mean values of the exercise factors, based on specification (4) in Table IV.

Standard factors	
Price-to-strike ratio	8
Volatility	−10
Dividend in next two weeks	3
Years to expiration	−10
Portfolio factors	
Correlation	23
Employee option portfolio	11
Additional factors	
Vesting date in past two weeks	37
Vesting date in past two weeks times years between prior two vesting dates	8
Price $\geq 90^{th}$ percentile prior-year distribution	50
Top-10 option holders/executive	−4
Male	10
Age	−5

Both of these variables are outside the set of factors that would be expected to be important either in a classical Black-Scholes setting or in a Black-Scholes setting augmented by portfolio factors.

Simulation Details

In a grant that vests in fractions $\alpha_1, \ldots, \alpha_n$ at times $t_1, \ldots, t_n$, the average option value (the cost of the option to the firm) is

$$O_t = \sum_{k=1}^{n} \alpha_k \mathrm{E}_t^* \left\{ \int_{t \vee t_k}^{T} e^{-r(\tau - t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s + \lambda) ds} \, d\tau \right.$$

$$\left. + e^{-r(T-t)} e^{-\int_t^T (G_s + \lambda) ds} (S_T - K)^+ \right\}.$$

To compute O_t for a given exercise function, G, and termination rate, λ, we use Monte Carlo simulation as follows:

1. Simulate a large number, N, of stock price paths between date 0 and option expiration date T at discrete time intervals of length Δt. For a given path i, we therefore simulate $J = \mathrm{T}/\Delta t$ values, where the j^{th} value is calculated as $S^i_{(j+1)\Delta \mathrm{t}} = S^i_{\mathrm{j}\Delta \mathrm{t}} \times e^{\left(r - \frac{1}{2}\sigma^2\right)\Delta \mathrm{t} + \sigma\sqrt{\Delta t}\epsilon^i_j}$ and where the ϵ^i_j are i.i.d. standard normal random variables.

Simulation Details

2. Along each path i, for each period j (corresponding to calendar date τ), calculate the probability that the option has survived to τ , $e^{-\int_t^{\tau}(G_s+\lambda)ds}$ and the conditional probability of the option being stopped this period, $(G_\tau + \lambda)\Delta t$,
   → $P(the\ option\ being\ stopped\ this\ period) = (G_\tau + \lambda)\Delta t \times e^{-\int_t^{\tau}(G_s+\lambda)ds}$

3. For each period, calculate the expected discounted cash flow conditional on the stock price path and stopping at time τ times the probability of stopping at time τ , $\mathrm{e}^{-\mathrm{r}(\tau-t)}(\mathrm{S}_\tau - \mathrm{K})^+(\mathrm{G}_\tau + \lambda)e^{-\int_t^{\tau}(G_s+\lambda)ds}\Delta t$
   and add these values for each period.

4. Also add in the expected cash flow at expiration (date T),
   $$\mathrm{e}^{-\mathrm{r}(T-t)}(\mathrm{S}_T - \mathrm{K})^+e^{-\int_t^{T}(G_s+\lambda)ds}$$

5. Repeat this for a large number of stock price paths, and average the results for each path to obtain an estimate of the option value, O_t.

05 Implications for Employee Option Cost to Firms

To implement the option valuation modeled in previous page,

$$O_t = \sum_{k=1}^{n} \alpha_k \mathrm{E}_t^* \left\{ \int_{t \vee t_k}^{T} e^{-r(\tau - t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s + \lambda) ds} \, d\tau \right.$$

$$\left. + e^{-r(T-t)} e^{-\int_t^T (G_s + \lambda) ds} (S_T - K)^+ \right\}.$$

we need an estimate of the employee termination rate λ as well as the voluntary exercise rate function $G(\boldsymbol{X}\beta)$.

Table VII

Summary Statistics for the Employee/Grant Cancellation Hazards by One-Digit SIC Code

Industry	One-Digit SIC Code	Mean Cancellation Hazard
Construction and Manufacturing	1 and 2	0.13
	3	0.12
Transportation, communications, and utilities	4	0.10
Retail	5	0.10
Finance, insurance, and real estate	6	0.04
Services	7	0.12
	8 and 9	0.15
Overall mean		0.107
Overall median		0.087

05 Implications for Employee Option Cost to Firms

A. Exercise Factor Effects in Employee Option Cost

Table VIII

Exercise Factor Effects in Employee Option Values

This table presents option values and percentage differences based on the estimated exercise functions presented in Tables IV and VI. All option values are for a 10-year at-the-money option with strike equal to 100, assuming the riskless interest rate is 5%, the employee termination rate is 10%, and the firm is in SIC 3. The base case assumes that stock return volatility is 30% and the dividend rate is 0, the stock return correlation with the market is 50%, and the option grant vests in year 2. Columns (1), (2), and (4) use coefficient estimates from specification (4) in Table IV with the standard, portfolio, and additional factors and top-10 holder fixed effects, and set employee risk, wealth, and age equal to their sample average values conditional on rank. Columns (6), (7), and (9) use coefficient estimates from specification (6) in Table VI with the standard, portfolio, and additional factors and top-10 holder interaction effects, and set employee risk, wealth, and age equal to their sample average values conditional on rank. Rank effect is the percentage difference in ESO value between top-10 and lower ranked option holders. Gender effect is the percentage difference in ESO value between male and female option holders.

	ESO Value with Std, Ptf, and Add'l Factors with Top-10 Fixed Effects					ESO Value with Std, Ptf, and Add'l Factors with Top-10 Interaction Effects				
	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)
Panel A: Vesting-Frequency Effects										
Cliff: Year 2	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
Annually: Years 1 to 4	29.89	30.37	2	31.04	−2	29.95	30.50	2	30.89	−1
Quarterly: Years 1 to 4	28.55	29.15	2	30.01	−3	28.62	29.90	4	30.37	−2
Monthly: Years 1 to 4	25.71	26.35	2	27.32	−4	25.78	27.67	7	28.24	−2
Vesting-frequency effect (%)	16	16		15		16	11		11	
Panel B: Volatility Effects										
0.20	24.64	25.18	2	25.93	−3	24.64	25.47	3	25.90	−2
0.30	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
0.40	36.82	37.33	1	38.02	−2	36.97	36.86	0	37.30	−1
0.50	42.53	43.02	1	43.68	−2	42.74	42.17	−1	42.63	−1
0.60	47.78	48.24	1	48.86	−1	48.04	47.05	−2	47.51	−1

05 Implications for Employee Option Cost to Firms

A. Exercise Factor Effects in Employee Option Cost

	ESO Value with Std, Ptf, and Add'l Factors with Top-10 Fixed Effects					ESO Value with Std, Ptf, and Add'l Factors with Top-10 Interaction Effects				
	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)
Panel C: Dividend Effects										
0.00	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
0.01	27.60	27.98	1	28.49	−2	27.65	27.92	1	28.24	−1
0.02	24.69	24.95	1	25.30	−1	24.73	24.92	1	25.14	−1
0.03	22.05	22.21	1	22.42	−1	22.07	22.20	1	22.33	−1
0.04	19.65	19.73	0	19.83	−1	19.66	19.72	0	19.79	0
0.05	17.48	17.50	0	17.50	0	17.49	17.49	0	17.50	0

Theory from Carpenter, Stanton, and Wallace (2010) gives a clear prediction that a higher dividend rate should accelerate exercise.

05 Implications for Employee Option Cost to Firms

B. Black-Scholes Approximations Currently Used in Practice

Since 2005, the FASB has required firms to recognize the cost of employee option compensation in their income statements.

- FAS 123R : a Modified Black-Scholes method that uses the Black-Scholes formula with the option's actual time to expiration replaced with its expected life
- SAB 110 : approves the use of the Black-Scholes formula with the option's stated maturity replaced by the average of the vesting date and stated maturity

Estimation for expected life:

- Assume that the option holder follows the exercise policy estimated in specification (4)
- The option's true expected term conditional on vesting is

$$L_t = E_t\left\{\int_{t\vee t_v}^{T} \tau(G_\tau+\lambda)e^{-\int_{t\vee t_v}^{\tau}(G_s+\lambda)ds}\,d\tau + Te^{-\int_{t\vee t_v}^{T}(G_s+\lambda)ds}\right\}$$

05 Implications for Employee Option Cost to Firms

B. Black-Scholes Approximations Currently Used in Practice

Table IX

Employee Option Values and Black-Scholes-Based Approximations

This table presents option values based on specification (4) in Table IV for a 52-year-old top-10 male option holder at a firm in SIC 3, labeled ESO value, and corresponding Black-Scholes approximations. BS to full term is the probability that the option vests times the Black-Scholes option value using the option's stated maturity. Midlife is the average of the option vesting date and stated maturity, and BS to midlife is the probability that the option vests times the Black-Scholes option value assuming expiration at the option's midlife. Exp term is the expected term of the option conditional on vesting, given the estimated exercise function and assumed termination rate. BS to exp term is the probability that the option vests times the Black-Scholes value of the option, assuming expiration at the option's expected term conditional on vesting. Implied term is the Black-Scholes term implied by the ESO value, conditional on vesting. Fitted term is the polynomial approximation of the implied term. BS to fitted term is the Black-Scholes value to this fitted term times the probability of vesting. The base-case option has a full term of 10 years, a strike price of \$100, and vests at year 2. The base case assumes an underlying stock price of \$100, 10% annual termination rate, 30% volatility, 0 dividend rate, 50% correlation with the market, 5% riskless rate, and 11% expected market return.

	ESO Value	BS to Full Term	Pct Err	Midlife	BS to Midlife	Pct Err	Exp Term	BS to Exp Term	Pct Err	Implied Term	Fitted Term	BS to Fitted Term	Pct Err
Panel A: Vesting Structure Effects													
Cliff: Year 2	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
Annual: 1 to 4	30.37	41.20	36	6.25	31.95	5	6.09	31.49	4	5.71	5.69	30.32	−0
Qrt: 1 to 4	29.15	42.79	47	6.06	32.63	12	5.42	30.63	5	4.97	4.96	29.15	−0
Mth: 1 to 4	26.35	43.15	64	6.02	32.78	24	4.61	28.16	7	4.10	4.10	26.33	−0
Panel B: Volatility Effects													
0.2	25.18	37.00	47	6.00	26.83	7	5.50	25.37	1	5.43	5.57	25.57	2
0.3	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
0.4	37.33	49.25	32	6.00	38.53	3	6.25	39.32	5	5.64	5.57	37.09	−1
0.5	43.02	55.11	28	6.00	44.22	3	6.48	45.80	6	5.65	5.57	42.71	−1
0.6	48.24	60.40	25	6.00	49.57	3	6.66	51.77	7	5.63	5.57	48.01	−0

05 Implications for Employee Option Cost to Firms

B. Black-Scholes Approximations Currently Used in Practice

	ESO Value	BS to Full Term	Pct Err	Midlife	BS to Midlife	Pct Err	Exp Term	BS to Exp Term	Pct Err	Implied Term	Fitted Term	BS to Fitted Term	Pct Err
Panel C: Dividend Rate Effects													
0.00	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
0.01	27.98	36.58	31	6.00	28.97	4	5.99	28.94	3	5.59	5.57	27.91	−0
0.02	24.95	30.94	24	6.00	25.63	3	6.04	25.70	3	5.63	5.57	24.83	−0
0.03	22.21	26.03	17	6.00	22.60	2	6.09	22.71	2	5.71	5.57	22.02	−1
0.04	19.73	21.77	10	6.00	19.86	1	6.14	19.98	1	5.85	5.57	19.46	−1
0.05	17.50	18.11	4	6.00	17.39	−1	6.20	17.49	−0	6.22	5.57	17.14	−2
Panel D: Termination-Rate Effects													
0.04	38.09	48.53	27	6.00	36.79	−3	6.78	39.37	3	6.38	6.36	38.00	−0
0.07	34.47	45.70	33	6.00	34.65	1	6.33	35.69	4	5.94	5.93	34.41	−0
0.10	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
0.13	28.52	40.53	42	6.00	30.73	8	5.60	29.57	4	5.26	5.26	28.52	−0
0.16	26.07	38.17	46	6.00	28.94	11	5.31	27.01	4	4.99	4.99	26.08	0

ESO value falls more slowly than the Modified Black-Scholes value as the dividend payout rate increases.

06 Conclusion

Find new exercise factor effects in option cost:

- vesting structure matters a lot—option cost drops 11% to 16% from cliff to monthly vesting since the passage of vesting dates triggers early exercises.
- top-ranked option holders exercise more slowly, with their options typically worth 2% to 7% more than those of lower ranked option holders.
- men exercise faster than women, reducing their options' value by around 2% to 4%.

Develop an analytical approximation of the option's implied Black-Scholes expiration date, which when incorporated into the Black-Scholes formula delivers values that much more closely approximate executive and employee option values across a wide range of parameters.